

MAR 20 1998

January 1998



Mathematics 30
Grade 12 Diploma Examination

Alberta
EDUCATION

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January 1998

Mathematics 30

Grade 12 Diploma Examination

Description

Time: 2.5 h. You may take an additional 0.5 h to complete the examination.

This is a **closed-book** examination consisting of

- 40 multiple-choice questions and 9 numerical-response questions, of equal value, worth 70% of the examination
- 3 written-response questions, of equal value, worth 30% of the examination

A tear-out formula sheet and a z-score page are included in this booklet.

All graphs on this examination are computer-generated.

Instructions

- Consider all numbers used in the questions to be **exact** numbers and not the result of a measurement.
- Fill in the information required on the answer sheet and the examination booklet as directed by the presiding examiner.
- You are expected to provide your own scientific calculator.
- Carefully read the instructions for each part before proceeding.
- The presiding examiner will collect your answer sheet and examination booklet and send them to Alberta Education.
- Do not fold the answer sheet.

Note: *The perforated pages at the back of this booklet may be torn out and used for your rough work. No marks will be given for work done on the tear-out pages.*

Multiple Choice

- Read each question carefully and decide which of the choices completes the statement or answers the question.
- Locate that question number on the separate answer sheet provided and fill in the circle that corresponds to your choice.

Example

This diploma examination is for
the subject of

- A. biology
B. physics
C. chemistry
D. mathematics

Answer Sheet

Ⓐ Ⓑ Ⓒ Ⓓ

- Use an **HB pencil only**.
- If you wish to change an answer, erase **all** traces of your first answer.

Numerical Response

- Read each question carefully.
- Record your answer on the answer sheet provided by writing it in the boxes and then filling in the corresponding circles.
- **Enter the first digit of your answer in the left-hand box and leave any unused boxes blank.**
- **Use an HB pencil only.**
- If you wish to change an answer, erase **all** traces of your first answer.

Sample Questions and Solutions

Example

Correct to the nearest tenth of a radian, 40° is equal to _____ rad.

$$40^\circ = 0.6981317008 \dots \text{rad}$$

**Record 0.7 on the
answer sheet —**

| | | | |
|---|---|---|---|
| 0 | . | 7 | |
| | ● | ● | |
| ● | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 |
| 7 | 7 | ● | 7 |
| 8 | 8 | 8 | 8 |
| 9 | 9 | 9 | 9 |

Example

For the arithmetic series $-8 + (-5) + (-2) + \dots + (85)$, the number of terms is _____.

$$85 = -8 + (n - 1)(3)$$

$$93 = 3n - 3$$

$$n = 32$$

Record 32 on the answer sheet →

| | | | |
|---|---|--|--|
| 3 | 2 | | |
|---|---|--|--|

| | |
|---|---|
| . | . |
| 0 | 0 |
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 4 |
| 5 | 5 |
| 6 | 6 |
| 7 | 7 |
| 8 | 8 |
| 9 | 9 |

Example

Place the following diploma examination subjects in alphabetical order.

- 1 biology
- 2 physics
- 3 chemistry
- 4 mathematics

(Record your answer as .)

Answer: 1342

Record 1342 on the answer sheet →

| | | | |
|---|---|---|---|
| 1 | 3 | 4 | 2 |
|---|---|---|---|

| | |
|---|---|
| . | . |
| 0 | 0 |
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 4 |
| 5 | 5 |
| 6 | 6 |
| 7 | 7 |
| 8 | 8 |
| 9 | 9 |

Written Response

- Read each question carefully.
- Write your answers in the examination booklet as neatly as possible.
- For full marks, your answers **must show all** pertinent explanations, calculations, and formulas.
- Your answers **should be** presented in a well-organized manner using complete sentences for a written response, and correct units for a numerical response.

British Literature

1. Introduction to British Literature

2. The Middle Ages

3. The Renaissance

4. The 17th Century

5. The 18th Century

6. The 19th Century

7. The 20th Century

8. Conclusion

British Literature

1. Introduction to British Literature

2. The Middle Ages

3. The Renaissance

4. The 17th Century

5. The 18th Century

6. The 19th Century

7. The 20th Century

8. Conclusion

1. Which of the following functions is a polynomial function?

A. $f(x) = 2^x + 6x^2 + 5$

B. $f(x) = x^2 + 6x + \sqrt{5}$

C. $f(x) = x^{\frac{2}{3}} + 5x^2 - 7$

D. $f(x) = \frac{x^2}{3} + \sqrt{5}x - \frac{1}{x}$

2. When the polynomial $x^3 - 5x^2 - 3x + 2$ is divided by $x - 1$, the quotient is

A. $x^2 - 6x + 3$

B. $x^2 - 6x - 3$

C. $x^2 + 4x + 1$

D. $x^2 - 4x - 7$

3. The equation of a polynomial function is $P(x) = kx(x + 2)(x - 6)$, $k \neq 0$. If a new function results from doubling the value of k only, then the zeros of the new function

A. remain unchanged

B. are doubled

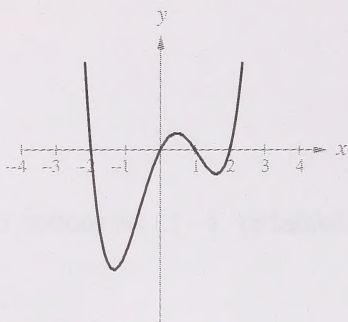
C. are 0, -1, and 3

D. are 0, -4, and 12

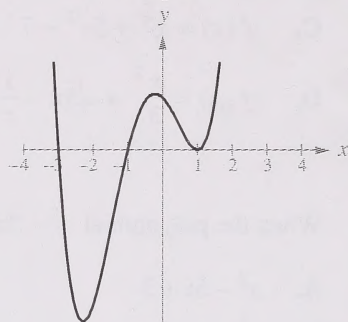
Use the following information to answer the next question.

The partial graphs of four **fourth-degree** polynomial functions are shown below. The x -intercepts are integers and **all** are shown.

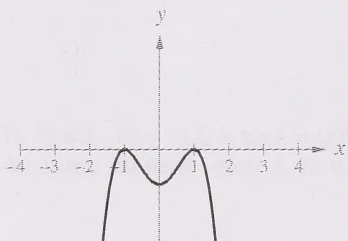
Graph 1



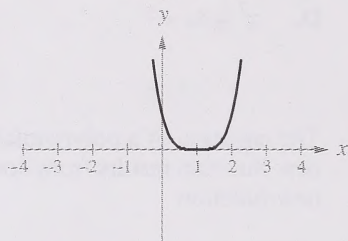
Graph 2



Graph 3

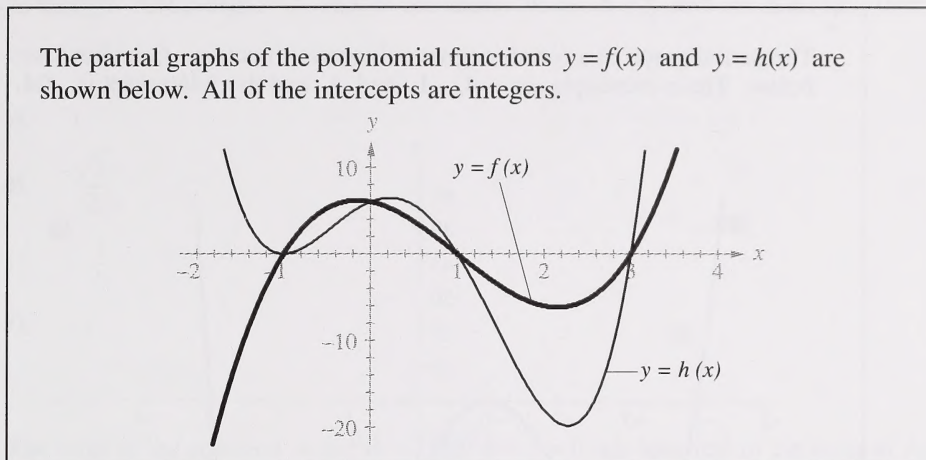


Graph 4



4. If the zeros of these functions are all integers, then the functions with a zero of multiplicity 2 at $x = 1$ are represented in
- A. graphs 1 and 4
 - B. graphs 2 and 3
 - C. graphs 1, 2, and 3
 - D. graphs 2, 3, and 4

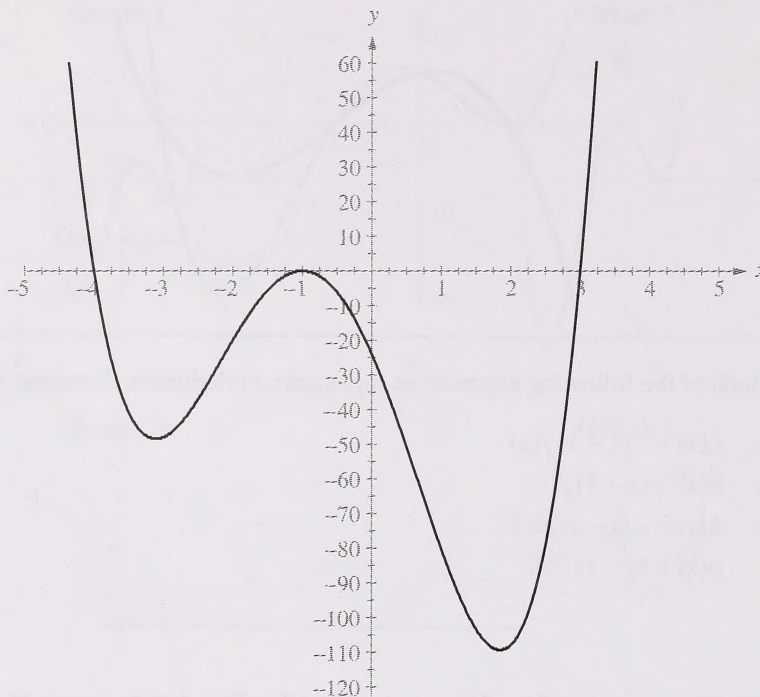
Use the following information to answer the next question.



5. Which of the following expressions represents a relationship between $f(x)$ and $h(x)$?
- A. $h(x) = -(x + 1)f(x)$
 - B. $h(x) = (x + 1)f(x)$
 - C. $h(x) = -(x - 1)f(x)$
 - D. $h(x) = (x - 1)f(x)$
-
6. A factor of $x^3 - x^2 - 3x + 3$ is $x - 1$. The other first-degree factors of this polynomial are
- A. $(x - 3)$ and $(x + 3)$
 - B. $(x - 3)$ and $(x - 3)$
 - C. $(x - 1.7)$ and $(x + 1.7)$
 - D. $(x - \sqrt{3})$ and $(x + \sqrt{3})$
7. A polynomial function, $P(x)$, has a remainder of 4 when it is divided by $x - 1$. If $2P(x)$ is divided by $x - 1$, then the remainder is
- A. 2
 - B. 4
 - C. 6
 - D. 8

Use the following information to answer the next question.

The partial graph of a fourth-degree polynomial function, f , is shown below. The x -intercepts are -4 , -1 , and 3 , and the y -intercept is -24 .



Numerical Response

1. If $f(x) = a(x + b)^2(x + c)(x - d)$, where a , b , c , and d are all **positive**, then

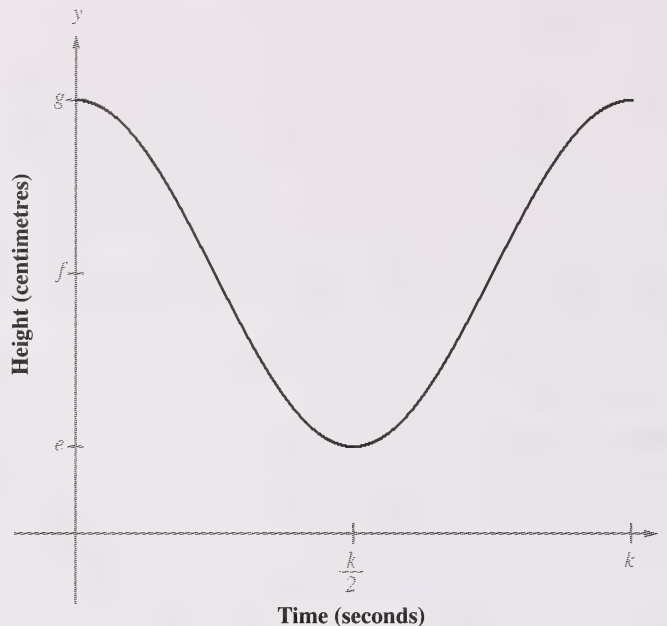
the numerical value of a is _____ (Record in column 1 of numerical response 1)
 the numerical value of b is _____ (Record in column 2 of numerical response 1)
 the numerical value of c is _____ (Record in column 3 of numerical response 1)
 the numerical value of d is _____ (Record in column 4 of numerical response 1)

(Record your answer as $\boxed{a}\boxed{b}\boxed{c}\boxed{d}$ in the numerical-response section of the answer sheet.)

8. In triangle ABC , angles A and B are acute. If $\cos A = \frac{1}{2}$ and $\sin B = \frac{\sqrt{3}}{2}$, then $\cos(A - B)$ is
- A. 1
 - B. $\frac{\sqrt{3}}{2}$
 - C. $-\frac{1}{2}$
 - D. -1
9. The roots of the equation $4 \sec^2 \theta - 7 \tan^2 \theta - 3 = 0$ are identical to the roots of the equation
- A. $7 \tan^2 \theta - 1 = 0$
 - B. $3 \tan^2 \theta - 1 = 0$
 - C. $3 \tan^2 \theta + 1 = 0$
 - D. $2 \sec \theta - \sqrt{7} \tan \theta - \sqrt{3} = 0$
10. The expression $\sin \theta \tan \theta + \cos \theta$, where $\cos \theta \neq 0$, is equal to
- A. $\cos \theta$
 - B. $\csc \theta$
 - C. $\sin \theta$
 - D. $\sec \theta$

Use the following information to answer the next question.

A clock pendulum released from its maximum height swings out and back to its starting position. A cosine function models the pendulum's height as a function of time. A student graphed one period of this function, as shown below.



11. Which of the following statements about this function could be true?

- A. The vertical displacement is e centimetres.
- B. The amplitude is $(g - e)$ centimetres.
- C. There is a phase shift of f seconds.
- D. The period is k seconds.

12. The range of $f(x) = a \sin(4x) - 2$, $a > 0$, is

- A. $-a \leq f(x) \leq a$
- B. $-2 \leq x \leq 2$
- C. $-a - 2 \leq f(x) \leq a - 2$
- D. $-a - 2 \leq x \leq a - 2$

13. The solutions of $2 \sin^3 \theta + 5 \sin^2 \theta - 3 \sin \theta = 0$, $0 \leq \theta < 2\pi$, are

- A. $\frac{\pi}{6}, \frac{5\pi}{6}$
- B. $0, \frac{\pi}{3}, \frac{2\pi}{3}$
- C. $0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$
- D. $\frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{2}$

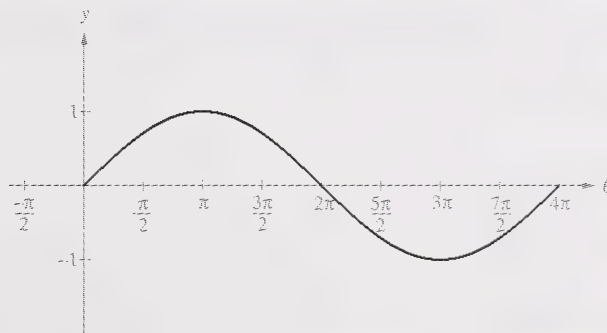
Numerical Response

2. An arc of length x metres subtends an angle of 60° at the centre of a circle with radius 1.0 m. The value of x , to the nearest hundredth of a metre, is _____ m.

(Record your answer in the numerical-response section of the answer sheet.)

Use the following information to answer the next question.

The graph of $y = \sin b\theta$, $0 \leq \theta \leq 4\pi$, is shown below. The θ -intercepts where $0 \leq \theta \leq 4\pi$, are 0, 2π , and 4π .



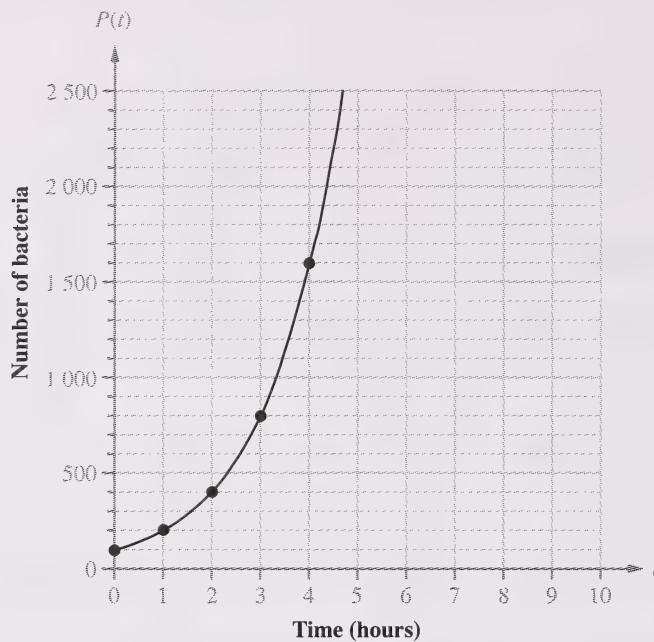
Numerical Response

3. Correct to the nearest tenth, the value of b is _____.

(Record your answer in the numerical-response section of the answer sheet.)

Use the following information to answer the next question.

In an experiment, the observed number of bacteria at time t , in hours, is graphed. A smooth curve is drawn through the points, as shown below.



14. If the approximate number of bacteria $P(t)$ at any time t is modelled by exponential growth, then $P(t)$ is given by
- A. $P(t) = 2(100)^t$
 - B. $P(t) = 100^{\frac{t}{2}}$
 - C. $P(t) = 2(100)^{2t}$
 - D. $P(t) = 100(2)^t$

15. The population of a small village is now 807, which is 3 times what it was 5 years ago. If the population continues to increase exponentially at this rate, then the population 7 years from now will be
- A. 1 560
 - B. 2 905
 - C. 3 757
 - D. 4 842
16. An exponential form of $n \log_a b = c$, $a > 1$, $b > 0$, and $n \neq 0$, is
- A. $a^c = b^n$
 - B. $a^b = c^n$
 - C. $b^a = c^n$
 - D. $b^c = a^n$
17. The point $(9, 4)$ lies on the graph of $y = 2 \log_b x$. Correct to the nearest tenth, the value of b is
- A. 0.5
 - B. 1.4
 - C. 2.0
 - D. 3.0

18. The value of the expression $\log_{\sqrt{5}} 5 + 2 \log_5 125$ is

- A. 11
- B. 9.5
- C. 8
- D. 6.5

19. If $14 = 7^{2x}$, then the value of x is

- A. $\frac{\log 7}{2}$
- B. $\frac{\log 2}{2}$
- C. $\frac{\log 14}{\log 49}$
- D. $\frac{\log 14}{\log \sqrt{7}}$

20. Atmospheric air pressure is halved for every 5.5 km increase in altitude above the surface of the Earth. The approximate air pressure at an altitude of 25 km, as a percentage of the surface air pressure, is

- A. 1.2 %
- B. 4.3 %
- C. 23 %
- D. 86 %

Numerical Response

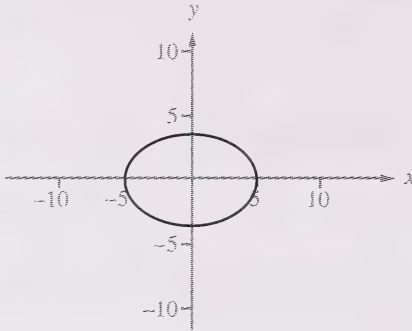
4. If $\log_b(a) = 3.8$ and $\log_b(c) = 2.1$, then correct to the nearest tenth, the value of $\log_b\left(\frac{a}{c}\right)$ is _____.

(Record your answer in the numerical-response section of the answer sheet.)

21. A **true** statement about the conic described by the equation $4x^2 - 6y^2 - 20 = 0$ is that it
- A. has a graph with y-intercepts that are -5 and 5
 - B. has an eccentricity between 0 and 1
 - C. is a locus of points equidistant from a fixed point
 - D. is the intersection of a plane with both nappes of a double-napped cone

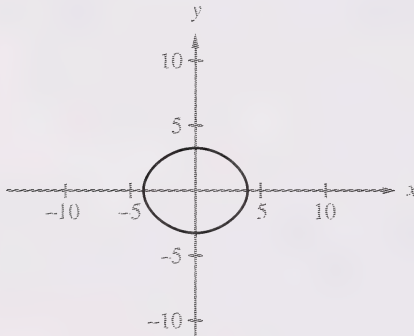
Use the following information to answer the next question.

The graph of $Ax^2 + Cy^2 + Dx + Ey + F = 0$, $F = -25$, is shown below.

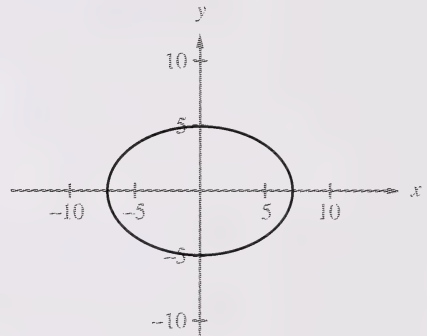


22. If the value of F is changed from -25 to -16 , and no other changes are made, the graph will look like

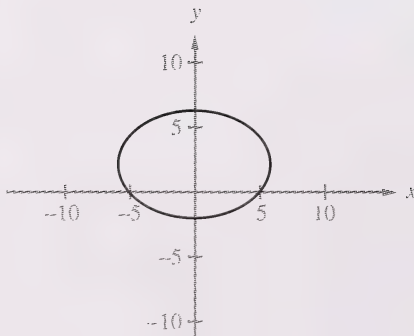
A.



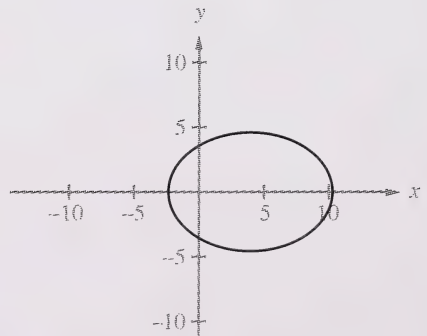
B.



C.

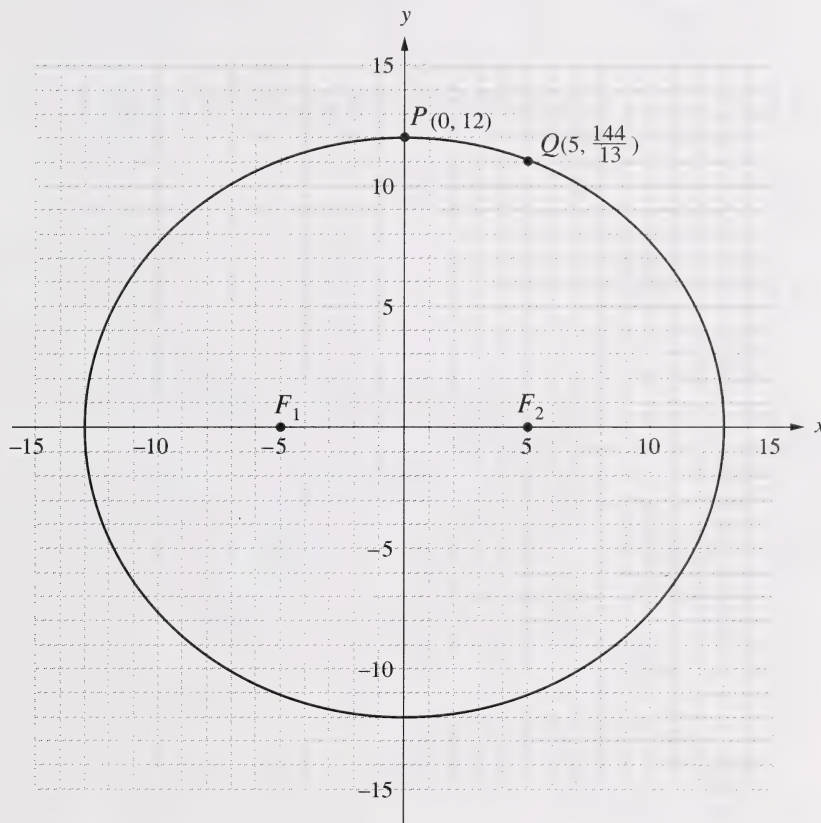


D.



Use the following information to answer the next question.

An ellipse centred at the origin with foci F_1 and F_2 at $(-5, 0)$ and $(5, 0)$, respectively, is shown below. The points $P(0, 12)$ and $Q\left(5, \frac{144}{13}\right)$ lie on the ellipse.



23. The value of $\overline{PF_1} + \overline{PF_2}$ is 26. Correct to the nearest tenth, the distance from point Q to F_1 is
- A. 49.0
 - B. 28.3
 - C. 14.9
 - D. 13.0

24. Chris substituted four different values for C into the equation $x^2 + Cy^2 + Dx + Ey + F = 0$. The other coefficients were kept the same. Each new equation determined a non-degenerate conic.

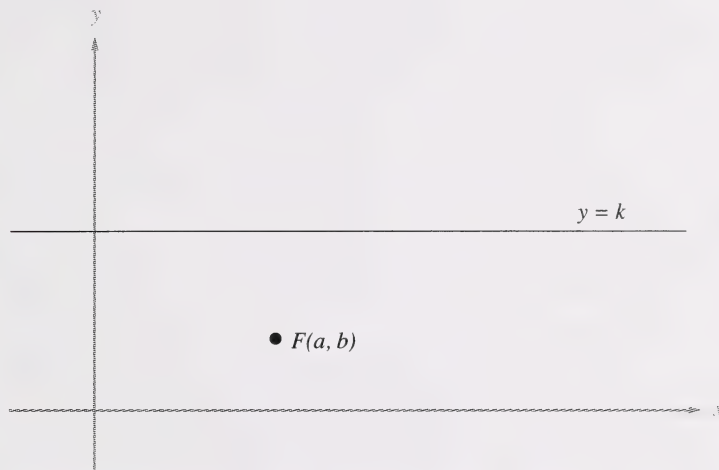
- The equation $x^2 + C_1y^2 + Dx + Ey + F = 0$ generated a hyperbola.
- The equation $x^2 + C_2y^2 + Dx + Ey + F = 0$ generated a parabola.
- The equation $x^2 + C_3y^2 + Dx + Ey + F = 0$ generated a circle.
- The equation $x^2 + C_4y^2 + Dx + Ey + F = 0$ generated an ellipse.

If C_1, C_2, C_3, C_4 form an arithmetic sequence, then the value of C_1 is

- A. -1
 - B. -2
 - C. -3
 - D. -4
25. A point P moves on a path in a plane such that its distance from point A is always 5 units. The path of P forms a
- A. pair of perpendicular lines
 - B. straight line
 - C. parabola
 - D. circle

Use the following information to answer the next question.

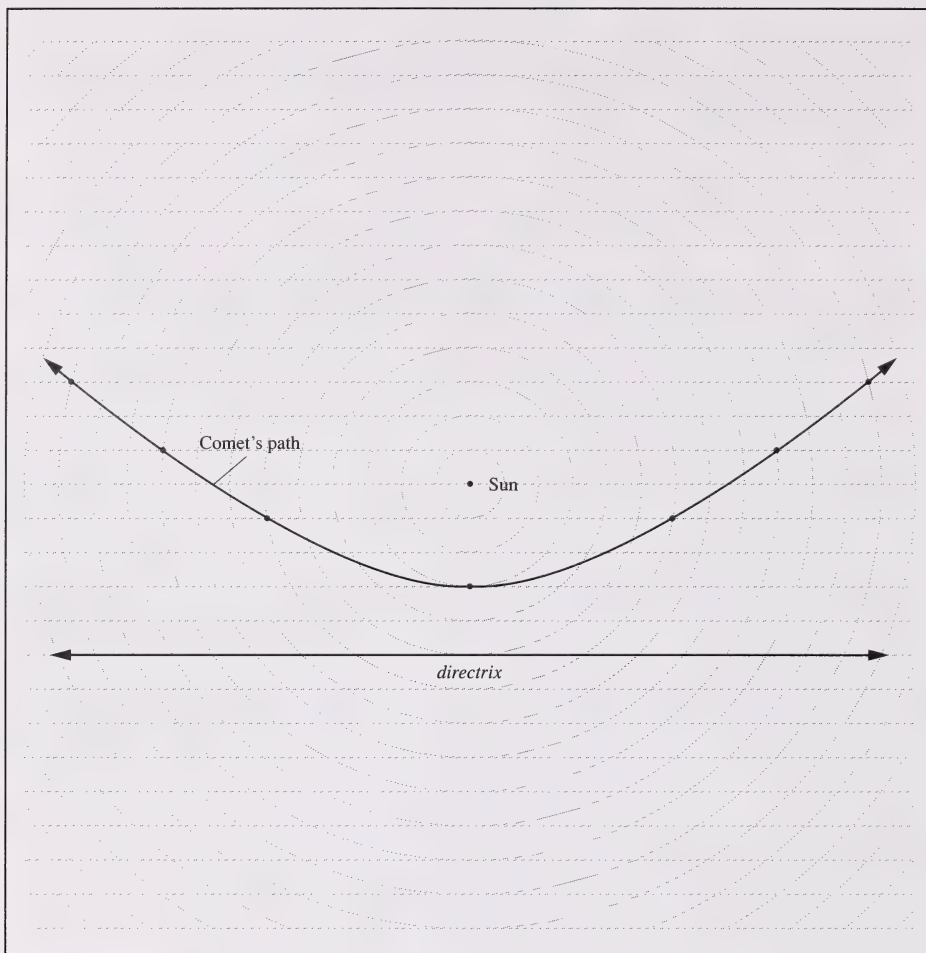
The graph of a parabola, $y = P(x)$, is used to model a relationship observed in an experiment. The directrix of the parabola is $y = k$, and the focus is $F(a, b)$, as shown below.



26. The maximum value of $P(x)$ is

- A. b
- B. k
- C. $\frac{k+b}{2}$
- D. $\frac{k-b}{2}$

Use the following information to answer the next question.



Numerical Response

5. The path of a comet is represented by a conic section with the Sun at a focus, as modelled above. The value of the eccentricity of this conic, correct to the nearest tenth, is _____.

(Record your answer in the numerical-response section of the answer sheet.)

27. The third term of the geometric sequence in which $t_1 = 64x^8$ and $t_{n+1} = \frac{t_n}{2x^2}$, $x \neq 0$, is
- A. $16x^2$
 - B. $16x^3$
 - C. $16x^4$
 - D. $32x^6$
28. The sum of the geometric series $\frac{1}{4} + \frac{1}{2} + 1 + \dots + 2\,048$ is
- A. 2 047.75
 - B. 2 049.75
 - C. 4 095.75
 - D. 4 096.75
29. The sum of an arithmetic sequence is given by $S_n = 2n^2 + 3n$. The common difference of this arithmetic sequence is
- A. $\frac{4}{5}$
 - B. 4
 - C. 9
 - D. 14

Use the following information to answer the next question.

The wavelengths of various types of electromagnetic radiation are given below, with the longest wavelength in each type of radiation shown. For example, the longest wavelength of a microwave is 4×10^{-1} m.

| Gamma waves | X-rays/ultraviolet | Visible/infrared | Microwaves |
|----------------|---------------------|--------------------|--------------------|
| | 4×10^{-10} | 4×10^{-7} | 4×10^{-4} |
| Wavelength (m) | | | |

30. The longest wavelength of each type of radiation listed in ascending order forms

- A. a geometric sequence with a common ratio of 10^3
- B. a geometric sequence with a common ratio of 3
- C. an arithmetic sequence with a common difference of 10^3
- D. an arithmetic sequence with a common difference of 3

31. Expressed in sigma notation, $\log_2 (2) + \log_2 (4) + \log_2 (8) + \log_2 (16)$ is equivalent to

- A. $\sum_{n=1}^4 (n)$
- B. $\sum_{n=1}^4 (2n)$
- C. $\sum_{n=1}^4 (n^2)$
- D. $\sum_{n=1}^4 (2^n)$

32. On the same day, Pat and Terry each start reading a historical novel that has 2185 pages. Pat reads 100 pages on the first day and 100 pages every day thereafter. Terry begins by reading 10 pages the first day and increases the number of pages read by 10 every day thereafter. On which day will Pat and Terry end their daily reading on the same page of the novel?
- A. 10th day
 - B. 19th day
 - C. 20th day
 - D. 21st day

Numerical Response

6. The first and second terms of an arithmetic sequence are 50 and 100, respectively. The first and second terms of a geometric sequence are also 50 and 100, respectively. The sum of the third terms of these two sequences is _____.

(Record your answer in the numerical-response section of the answer sheet.)

33. A tune of "*Row, Row, Row Your Boat* " has 5 notes in its first line:

C C C D E

Assume that all 5 notes are held for the same length of time. If the notes are rearranged at random, how many different melodies could be composed?

- A. 6
- B. 20
- C. 40
- D. 120

34. How many different arrangements consisting of three letters, beginning and ending with different consonants and having a vowel in the middle, can be made with the letters of the word **R E D I A L**?
- A. 18
 - B. 20
 - C. 27
 - D. 120
35. A committee of 7 people is to be chosen from a city council that consists of a mayor, a deputy mayor, and 13 councillors. The mayor and deputy mayor must be on the committee, and because of a conflict of interest, three councillors cannot be on the committee. The number of committees, given these restrictions, that can be chosen from this city council is
- A. $_{10}C_5$
 - B. $_{12}C_4$
 - C. $_{13}C_2$
 - D. $_{15}C_7$
36. A jar of 12 lollipops contains an equal number of red, green, purple, and yellow lollipops. Without looking, a person reaches into the jar and removes a handful of 4 lollipops. The probability that the handful of lollipops includes one lollipop of each colour is
- A. $\frac{1}{495}$
 - B. $\frac{1}{11\,880}$
 - C. $\frac{9}{55}$
 - D. $\frac{3}{440}$

Numerical Response

7. Correct to the nearest whole number, the value of $\sum_{n=1}^8 {}_7C_{(n-1)}$ is _____.

(Record your answer in the numerical-response section of the answer sheet.)

Numerical Response

8. Nicole misplaced a 7-digit phone number. She knows that the phone number begins with 4 and the last six digits are 1, 2, 3, 5, 7, and 8, in some order. The number of phone numbers that satisfy these conditions is _____.

(Record your answer in the numerical-response section of the answer sheet.)

37. In the expansion of $(x - 2)^8$, the numerical coefficient of the term of degree 5 is
- A. 2 688
 - B. 448
 - C. -448
 - D. -2 688

Use the following information to answer the next question.

Alex wrote four diploma examinations. From his study of statistics, he knows that a comparison of relative performance on the four different examinations is determined by the calculation of z -scores. The exam results are shown in the table below.

| Subject | Provincial Mean | Standard Deviation | Alex's Score |
|----------------|-----------------|--------------------|--------------|
| Chemistry 30 | 62.1 | 16.0 | 65 |
| English 30 | 64.4 | 12.8 | 80 |
| Mathematics 30 | 62.5 | 17.4 | 83 |
| Physics 30 | 66.6 | 17.1 | 77 |

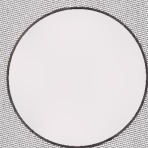
38. Alex performed **best**, relative to other students in the province, in
- A. Chemistry 30
 - B. English 30
 - C. Mathematics 30
 - D. Physics 30
39. A small airline has determined that the mass of luggage carried by passengers boarding an aircraft is normally distributed with a mean of 15 kg and a standard deviation of 2.7 kg. The percentage of passengers that will have luggage with a mass less than 12 kg is
- A. 11.10%
 - B. 13.35%
 - C. 36.65%
 - D. 38.90%
40. The length of time that light bulbs will be functional is normally distributed with a mean of 800 hours and a standard deviation of 160 hours. If testing determines that 0.3% of the bulbs fail within the first x hours, then x in hours is
- A. 360
 - B. 422
 - C. 480
 - D. 717

Numerical Response

- 9.** The marks on a mathematics examination are normally distributed with a mean of 60% and standard deviation of 10%. Correct to the nearest hundredth, the probability, expressed as a decimal, that a student scores between 50% and 70% on this examination is _____.

(Record your answer in the numerical-response section of the answer sheet.)

The written-response questions follow on the next page.

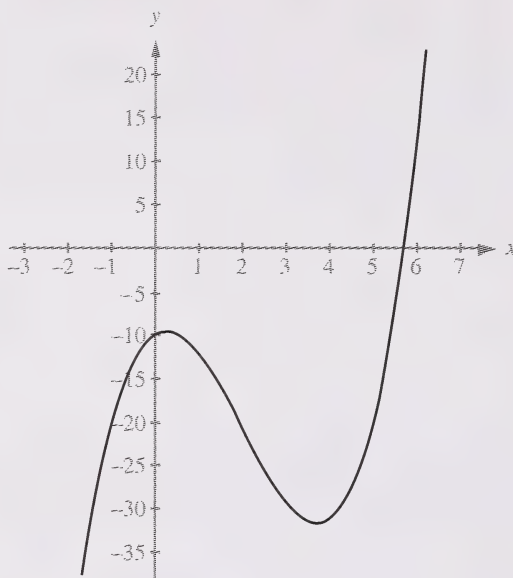


Written Response — 10%

1. A student was attempting to graph $y = P(x)$, where

$$P(x) = x^3 - 6x^2 + 3x + 10.$$

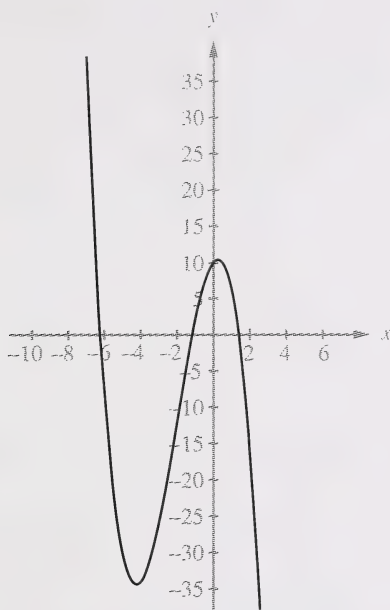
- The student entered an equation on the computer. The resulting graph of the equation is shown below.



This graph does **not** represent $y = x^3 - 6x^2 + 3x + 10$ because the student entered **one** term of the equation incorrectly. Which term was entered incorrectly? By referring to the graph above, explain how you reached your conclusion.

Written-response question 1 continues on the next page.

- The student **again** attempted to graph $y = x^3 - 6x^2 + 3x + 10$ and obtained the following graph.



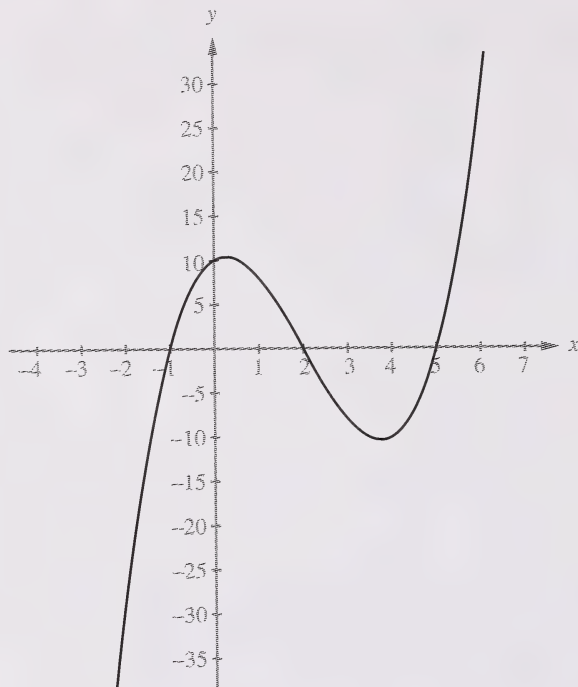
This second graph does **not** represent the graph of $P(x)$ because the student again entered **one** term of the equation incorrectly. Which term was entered incorrectly this time? By referring to the graph above, explain how you reached your conclusion.

Written-response question 1 continues on the next page.

- Finally, the student entered the equation

$$y = x^3 - 6x^2 + 3x + 10$$

correctly into the computer. The resulting graph is shown below.

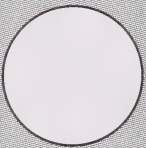


From this graph, estimate and list the x -intercepts. Write the factored form of $P(x)$.

Written-response question 1 continues on the next page.

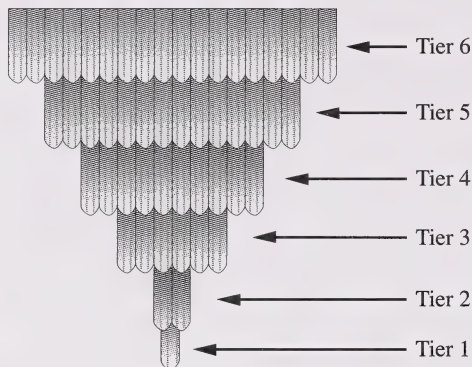
- Algebraically verify that your factored form of $P(x)$ is correct.

Written-response question 2 begins on the next page.



Use the following information to answer the next question.

A lighting store specializes in custom-made chandeliers built in tiers. Each tier consists of a number of rectangular glass pieces arranged in a circular formation.



The chandelier shown is not the exact one described in this written-response question.

Written Response — 10%

2. The numbers of pieces of glass in each successive tier of a chandelier form a geometric sequence. Every chandelier uses 1 glass piece for tier 1. In a chandelier with 5 or more tiers, there are 81 glass pieces in tier 5.

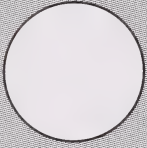
- Complete the chart below by indicating the number of glass pieces required for tiers 2, 3, and 4, and by indicating the total number of glass pieces required for a chandelier consisting of 2, 3, 4, or 5 tiers.

| Tier Number (n) | Number of glass pieces in n^{th} tier | Total number of glass pieces in a chandelier with n tiers |
|---------------------|--|---|
| 1 | 1 | 1 |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | 81 | |

Written-response question 2 continues on the next page.

- The cost for each glass piece is \$1.75. Determine the cost of the glass pieces required for a 7-tiered chandelier.
- Write a general expression of the total cost of the glass pieces required for a chandelier of n tiers, where $n \in N$.
- The manufacturer of the glass pieces has determined that the width of each piece is normally distributed about a mean of 4.00 cm with a standard deviation of 0.05 cm. Any piece with a width less than 3.90 cm or more than 4.10 cm cannot be used for a chandelier. If 100 000 glass pieces are selected at random for chandeliers, how many pieces from this initial selection will not meet the size requirement?

Written-response question 3 begins on the next page.



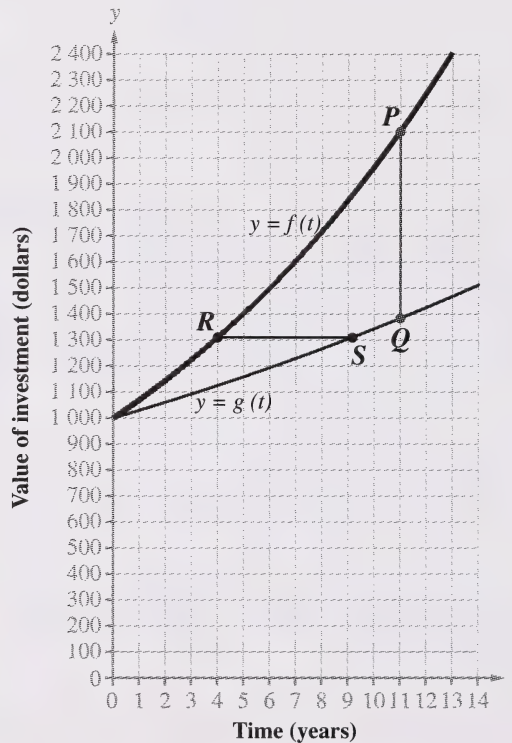
Use the following information to answer the next question.

Student A and student B each deposited \$1 000 in different investment plans.

The partial graph of the exponential function f approximates the value of the \$1 000 that student A deposited into a plan earning 7%/a interest compounded annually.

The partial graph of the exponential function g approximates the value of the \$1 000 that student B deposited into a plan earning $i\%/a$ interest compounded annually.

Both graphs are shown below.



Written-response question 3 continues on the next page.

Written Response — 10%

- 3.** The function $f(t) = 1\,000(1.07)^t$, $t \in N$, represents the value in dollars, after t years, of the \$1 000 that student A invested at 7%/a compounded annually.

- Determine algebraically the minimum amount of time it will take for student A's investment to be worth at least \$2 100.

Explain how the graph supports or does not support your answer.

Written-response question 3 continues on the next page.

- The point $(2, 1\,060.90)$ lies on the graph of $y = g(t)$.
Algebraically determine the interest rate.

- By referring to the **two** investments described in the problem,
explain what the length of the vertical line segment \overline{PQ} and the
length of the horizontal line segment \overline{RS} represent.

You have now completed the examination.

If you have time, you may wish to check your answers.

Mathematics 30 Formula Sheet

The following information may be useful in writing this examination.

- The roots of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The distance between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Quadratic Relations

- $e = \frac{|\overline{PF}|}{|\overline{PD}|}$

Trigonometry

- arc length $a = r\theta$
- $\sin^2 A + \cos^2 A = 1$
- $1 + \tan^2 A = \sec^2 A$
- $1 + \cot^2 A = \csc^2 A$
- $\sin(A + B) = \sin A \cos B + \cos A \sin B$
- $\sin(A - B) = \sin A \cos B - \cos A \sin B$
- $\csc A = \frac{1}{\sin A}$
- $\sec A = \frac{1}{\cos A}$
- $\cot A = \frac{\cos A}{\sin A}$
- $\cos(A + B) = \cos A \cos B - \sin A \sin B$
- $\cos(A - B) = \cos A \cos B + \sin A \sin B$

Permutations and Combinations

- ${}_nP_r = \frac{n!}{(n-r)!}$
- ${}_nC_r = \frac{n!}{r!(n-r)!}$
- In the expansion of $(x + y)^n$, the general term is $t_{k+1} = {}_nC_k x^{n-k} y^k$

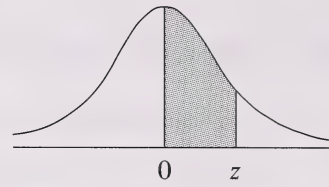
Sequences and Series

- $t_n = a + (n-1)d$
- $S_n = \frac{n[2a + (n-1)d]}{2}$
- $S_n = n\left(\frac{a + t_n}{2}\right)$
- $t_n = ar^{n-1}$
- $S_n = \frac{a(r^n - 1)}{r - 1}, r \neq 1$
- $S_n = \frac{rt_n - a}{r - 1}, r \neq 1$

Exponential and Logarithmic Functions

- $\log_a mn = \log_a m + \log_a n$
- $\log_a \frac{m}{n} = \log_a m - \log_a n$
- $\log_a m^n = n \log_a m$

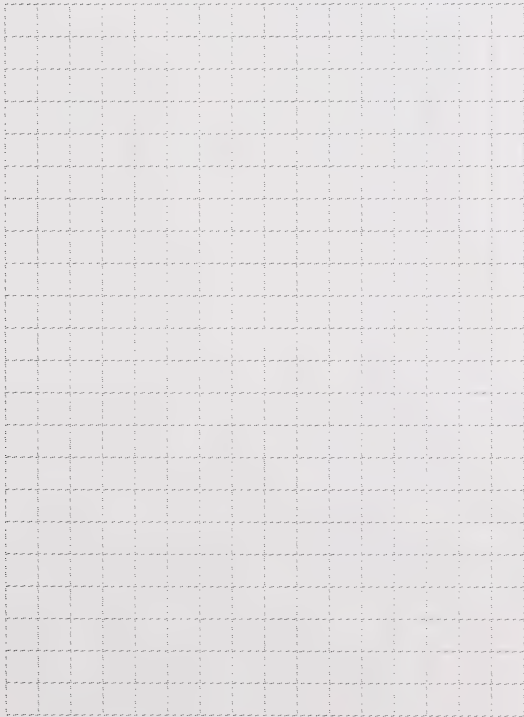
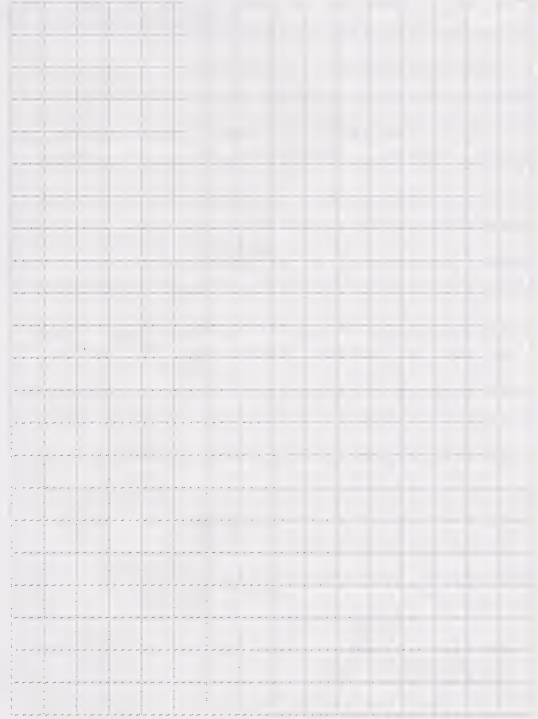
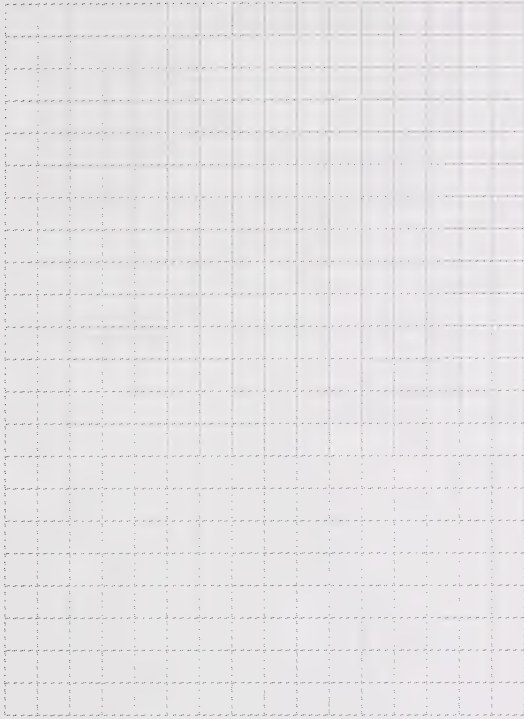
$$z = \frac{x - \mu}{\sigma}$$



Areas under the Standard Normal Curve

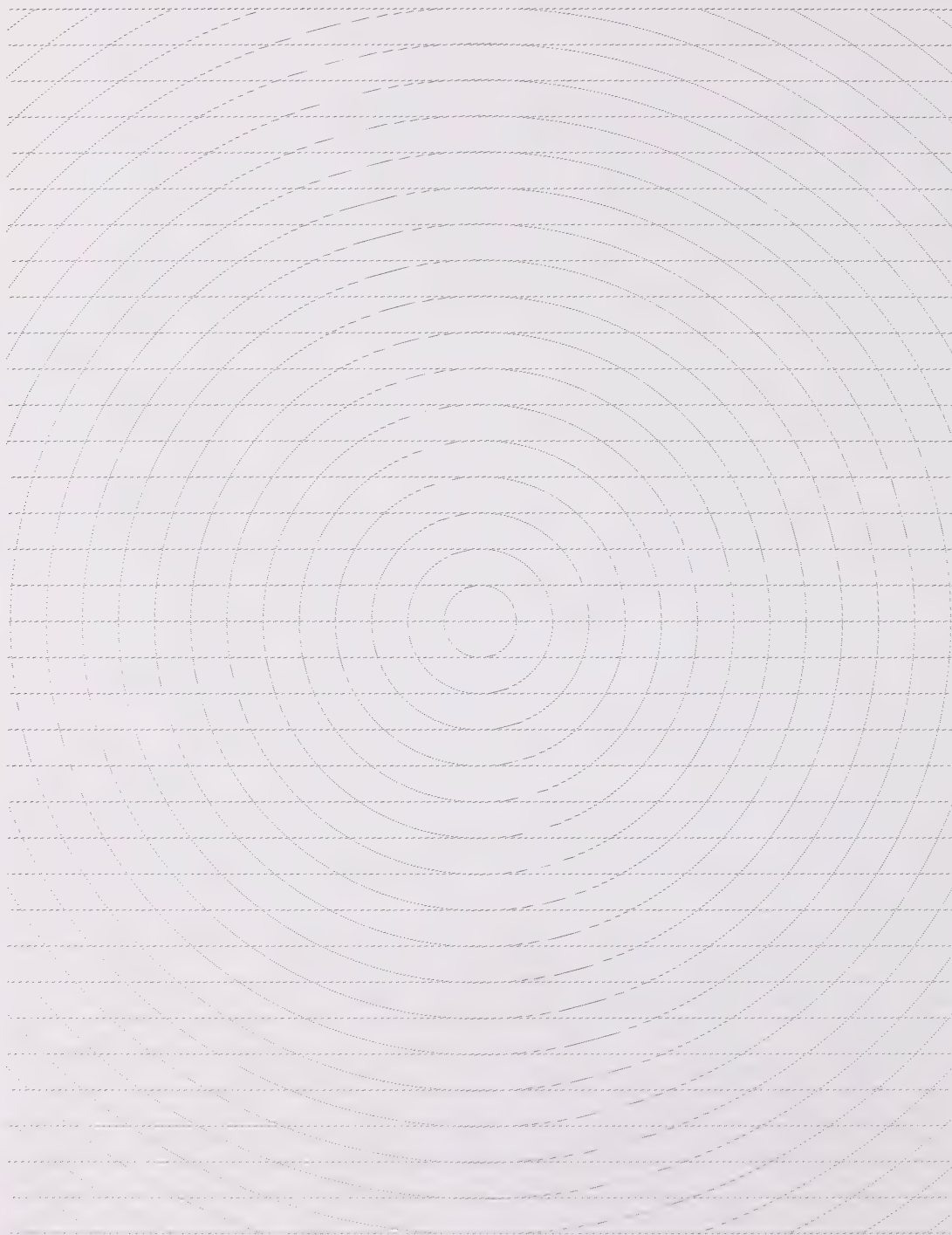
| <i>z</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0754 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2258 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2518 | 0.2549 |
| 0.7 | 0.2580 | 0.2612 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2996 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |
| 3.1 | 0.4990 | 0.4991 | 0.4991 | 0.4991 | 0.4992 | 0.4992 | 0.4992 | 0.4992 | 0.4993 | 0.4993 |
| 3.2 | 0.4993 | 0.4993 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4995 | 0.4995 | 0.4995 |
| 3.3 | 0.4995 | 0.4995 | 0.4995 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4997 |
| 3.4 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4998 |
| 3.5 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 |
| 3.6 | 0.4998 | 0.4998 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.7 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.8 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.9 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 |

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